SPACE DOMAIN DECOUPLING OF LSE AND LSM FIELDS IN GENERALIZED PLANAR GUIDING STRUCTURES

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Summary

LSE and LSM fields in generalized planar guiding structures are shown to be coupled only by means of the edge condition, which can be fulfilled as a final step in the analysis. This is utilized by applying the singular integral equation method to fin-lines. Modes up to the 20th are thus easily computed.

INTRODUCTION

In generalized planar guiding structures, thin metal strips are printed at air-dielectric or dielectric-dielectric interfaces as shown in Fig. 1. If these strips are assumed to be infinitesimally thin and perfectly conducting, LSE and LSM fields can be excited independently. This will be shown in the first part of our contribution. This decoupling between both parts of the general field offers great advantages for the analysis in the space domain as is illustrated in the second part by applying the singular integral equation technique to fin-lines.

DECOUPLING OF LSE AND LSM MODES

Assuming propagation in z-direction (Fig. 1), the LSE field can be expressed in terms of 2 scalar potentials in sub-regions 1 and 2, which must satisfy the scalar Helmholtz equation.

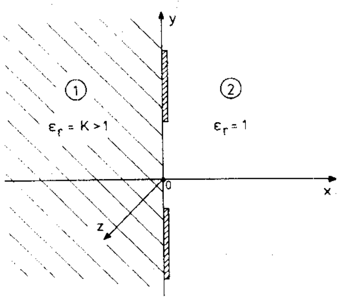

Fig. 1 Generalized planar guiding structure.

The interface conditions at the (x=0)-plane are $E_y^{(1)} = E_y^{(2)} = E_y$, $E_z^{(1)} = E_z^{(2)}$ and $H_z^{(1)} - H_z^{(2)} = J_y$, $H_y^{(2)} - H_y^{(1)} = J_z$ with $\underline{J}_s = J_y\underline{e}_y + J_z\underline{e}_z$ the surface current density and $\underline{e}_y$, $\underline{e}_z$ unit vectors in y- and z-direction, respectively.

One can prove from the relations between the transverse field components and the potentials that

$$E_z \sim dE_y/dy; \quad J_z \sim dJ_y/dy. \qquad (1)$$

Hence the LSE field is completely characterized by E_y and J_y.

Another important relation follows from the same equations: Both $\underline{E}_t = E_y\underline{e}_y + E_z\underline{e}_z$ and $\underline{J}_s$ have zero transverse divergence.

$$\underline{\nabla}_t \cdot \underline{E}_t = 0; \quad \underline{\nabla}_t \cdot \underline{J}_s = 0. \qquad (2)$$

$\underline{\nabla}_t$ means the grad-operator in the y-z-plane.

A similar analysis holds for the LSM field. The interface conditions at the (x=0)-plane are of course the same as above. Studying the relation between the field components and the scalar potential functions one finds

$$E_y \sim dE_z/d_y; \quad J_y \sim dJ_z/d_y \qquad (3)$$

instead of (1) and

$$\underline{n} \cdot (\underline{\nabla} \times E_t) = 0; \quad \underline{n} \cdot (\underline{\nabla} \times J_s) = 0 \qquad (4)$$

instead of (2). $\underline{n}$ means unit vector in x-direction (i.e. normal to the air-dielectric interface).

Eqs. (3) show that the LSM field is completely characterized by E_z and J_z, while eqs. (4) state that both $\underline{E}_t$ and $\underline{J}_s$ have zero normal curl.

One can summarize that the LSE as well as the LSM field alone fulfill the interface conditions with (2) and (4) being additional characteristic features. (Nevertheless they have been assumed to be coupled in all respective papers which are known to the authors). The edge condition /1/ will, however, establish a coupling between the LSE and the LSM fields. This can be proven in the following way: Both E_y and J_z are singular at the edges of the strips whereas E_z and J_y are non-singular here (namely they are vanishing). For the LSE field eqs. (1) hold. Taking in mind that the derivative of a function is stronger singular than the function itself, one recognizes that (1) is in contradiction to the edge condition as far as E_z

is concerned. The same conclusion holds for a LSM field concerning the surface current component J_y. Hence a linear combination of LSE and LSM fields must be used in order to fulfill the edge condition. In fact the LSE part in this combination is responsible of the J_z-singularity whereas the LSM part is responsible of the E_y-singularity. Such a coupling of both parts of the total field can, however, be performed as final step of the analysis, i.e. both the LSE and the LSM field can be treated independently up to this step.

An independent analysis of LSE and LSM fields has already been performed in the spectral domain, /2/ - /8/. It has been shown there to be very efficient. In /2/ - /5/ both LSE and LSM parts have been coupled in the spectral domain as a final step in the analysis. The necessity of such a coupling has not been justified. In /6/ - /8/ the coupling has been introduced in order to eliminate a matrix singularity. A physical reason could not be given.

The decoupling of LSE and LSM fields in the sense defined above is still more important for space domain techniques, in which the problem is described by integral rather than by algebraic equations. Dealing with uncoupled integral equations facilitates the work to a large extent in particular for planar structures with multi-layer dielectric.

SINGULAR INTEGRAL EQUATION TECHNIQUE

One of the most powerful space domain techniques is the singular integral equation technique, which has been used in /9/ - /11/ in solving many waveguide problems. It has also been used in /12/ for the analysis of microstrip lines. Using this technique for an analysis of generalized planar structures has the same advantages as the well-known and widely used Galerkin method in the spectral domain: the small order of the matrix characterizing the problem. For the dominant and the first few higher -order modes of any planar guiding structure, the Galerkin method in spectral domain is superior over all other methods, because the order of the matrix may be as low as 4 (corresponding to 2 basis functions for each component of either strip current or slot electric field) for still excellent accuracy. For higher-order modes up to the 10th or 20th, which are needed in the analysis of discontinuities, 2 basis functions are not sufficient to approximate the real field, so that the order of the matrix must be increased considerably. In this case the singular integral equation method becomes preferable, because a matrix of order 7 is quite sufficient for achieving accurate results up to the 30th mode.

Utilizing the decoupling between LSE and LSM fields derived above, the singular integral equation method leads to 2 uncoupled singular integral equations which are solved straightforward. In a subsequent final step the coupling is then taken into account in order to fulfill the edge condition. The procedure will be illustrated by analyzing the structure sketched in Fig. 2.

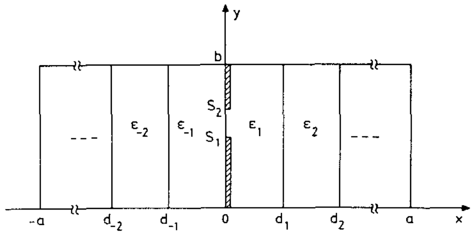

Fig. 2 Generalized unilateral fin-line.

At the interface at x=0, four functions are defined via

$$\frac{df_1^e}{dy} \sim \underline{\nabla}_t \underline{E}_t, \; f_2^e \sim (\underline{\nabla}_t \underline{J}_s),$$

$$f_1^h \sim (\underline{n}\cdot\underline{\nabla}\times\underline{E}_t), \; \frac{df_2^h}{dy} \sim \underline{n}\cdot\underline{\nabla}\times\underline{J}_s .$$

Comparing with (2) and (4) it can be seen that the first two represent the LSM part of the field while the latter two represent the LSE part. These functions can be expanded into Fourier series with two sets of unknown coefficients. The boundary conditions

$$\frac{df_1^e}{dy}=0=f_1^h \text{ on the fins, } f_2^e=0=\frac{df_2^h}{dy} \text{ in the slot}$$

result in two singular integral equations which are solved in terms of two infinite series with exponentially vanishing coefficients. The series are truncated behind the N-th term so that (2N) equations are obtained /11/. One recognizes that the boundary condition on the fins guarantees only that E_z and E_y satisfy Laplace equation. Hence E_z or E_y must be set to zero at the edges in order to fulfill $\underline{E}_t=0$ on the fins. Thus one coupling relation between the LSE and LSM fields is established. Similarly a second coupling relation arises by equating one of the components of the surface current density to zero at the edges of the fins. These 2 equations together with the other (2N) equations obtained from the standard solution of the 2 singular integral equations constitute a homogeneous system of linear equations, from which the propagation constants and the field distributions of the different modes are obtained.

NUMERICAL RESULTS

To illustrate the fast convergence of the truncation, the propagation constants of the first modes of a bilateral fin-line have been calculated for a matrix order of 5, 7, and 9 (i.e. truncating the infinite series behind the 2nd, 3rd, and 4th term, respectively). The results in Table 1 show that orders of 7 and 9 nearly give the same propagation constants. The dispersion characteristics of the first two modes in a bilateral fin-line are shown in Fig. 3 and for another slot width in Fig. 4. Circles and crosses mark results taken from /13/. The agreement is very good. The same holds for the plots of ε_{eff} versus slot width w (Fig. 5). The results to be compared with have now been taken from /4/.

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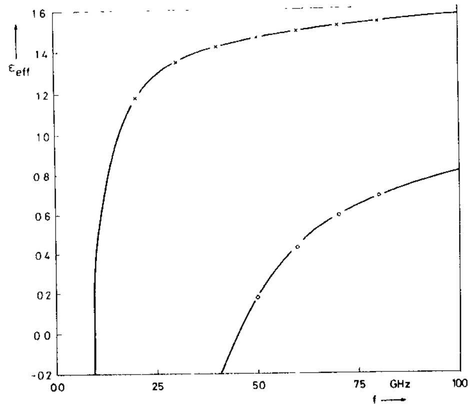

Fig. 3 Bilateral fin-line

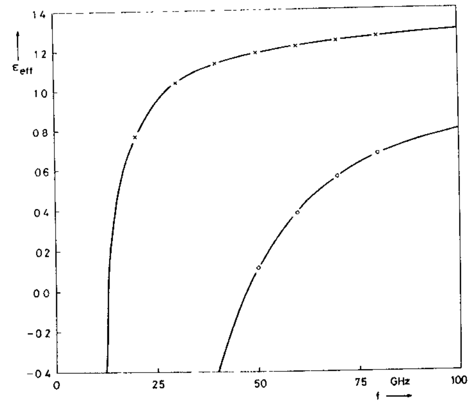

Fig. 4 Bilateral fin-line

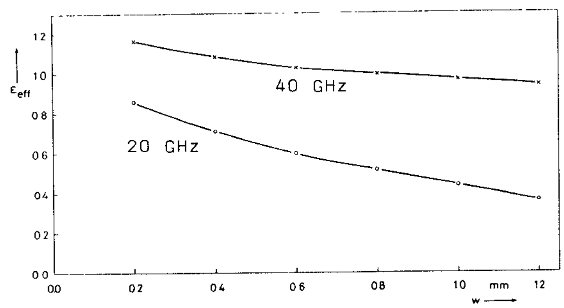

Fig. 5 Unilateral fin-line

matrix order \ modes	1	2	3	4	5	6	7	8	9	10
5 x 5	0.665	-j0.769	-j1.672	-j1.763	-j1.889	-j2.004	-j2.467	-j2.536	-j2.731	-j3.206
7 x 7	0.658	-j0.772	-j1.672	-j1.763	-j1.889	-j2.007	-j2.467	-j2.537	-j2.732	-j3.206
9 x 9	0.656	-j0.773	-j1.672	-j1.763	-j1.889	-j2.008	-j2.467	-j2.538	-j2.732	-j3.206

Table 1: The propagation constants of the first 20 modes in a bilateral fin-line.

matrix order \ modes	11	12	13	14	15	16	17	18	19	20
5 x 5	-j3.262	-j3.494	-j3.596	-j3.605	-j3.687	-j3.931	-j3.961	-j4.019	-j4.114	-j4.432
7 x 7	-j3.262	-j3.494	-j3.596	-j3.605	-j3.687	-j3.931	-j3.961	-j4.019	-j4.112	-j4.432
9 x 9	-j3.262	-j3.494	-j3.596	-j3.605	-j3.687	-j3.931	-j3.961	-j4.019	-j4.112	-j4.432